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DETERMINATION OF UPWASH AROUND A BODY OF REVOLUTION AT SUPERSONIC VELOCITIES

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INTRODUCTION:

THE METHOD OF ANALYSIS PRESENTED HEREAFTER IS A STRAIGHTFORWARD APPLICATION OF THE FORMULAS FOR THE POTENTIAL FLOW AROUND BODIES OF REVOLUTIONS AT SUPERSONIC VELOCITIES WHICH HAVE BEEN ESTABLISHED BY H. S. TSIEN (JOURNAL OF THE AERONAUTICAL SCIENCES, OCTOBER 1938: SUPERSONIC FLOW OVER AN INCLINED BODY OF REVOLUTION).

DERIVATION OF THE FORMULAS:

CONSIDER A BODY OF REVOLUTION, THE AXIS OF WHICH IS INCLINED AT AN ANGLE TO THE DIRECTION OF THE AIRSTREAM. THE PERTURBATION VELOCITIES DUE TO THE PRESENCE OF THE BODY CAN BE DEFINED — IN THE RANGE OF VALIDITY OF THE HYPOTHESES FOR WHICH THE FORMULAS DERIVED BY H. S. TSIEN ARE VALID — BY THE DERIVATIVES OF A PERTURBATION POTENTIAL ϕ_2 OF THE FORM:

$$\phi_2 = -\beta \cos\theta \int_{\cosh^{-1}\frac{X}{\beta r}}^{0} f(X-\beta r \cosh u) \cosh u \, du$$

WHERE:

X, r, θ = CYLINDRICAL COORDINATES

$\beta = \sqrt{M^2-1}$, WHERE M IS THE MACH NUMBER OF THE FLOW

u = PARAMETER OF INTEGRATION

THE INVESTIGATION IS LIMITED HERE TO THE MERIDIAN PLANE PERPENDICULAR TO THE PLANE OF SYMMETRY OF THE FLOW, AND WHICH CORRESPONDS TO $\theta = 0$. THE DOWNWASH VELOCITY IS NORMAL TO THE PLANE $\theta = 0$ AND HAS THE VALUE:

$$W = \frac{\phi_2}{r} = -\frac{\beta}{r} \int_{\cosh^{-1}\frac{X}{\beta r}}^{0} f(X-\beta r \cosh u) \cosh u \, du$$

APPLICATION TO CONICAL AND CYLINDRICAL FUSELAGES:

IN THE CASE OF A CONE OF REVOLUTION, IT HAS BEEN SHOWN BY H. S. TSIEN THAT THE POTENTIAL ϕ_2 CAN BE WRITTEN IN THE FORM:

$$\phi_2 = \frac{1}{2} K\beta^2 r \cos\theta \, \Psi\left(\frac{X}{\beta r}\right)$$

THE CONSTANT K BEING DEFINED BY THE CONDITION

$$v = \alpha U = -\frac{1}{2} K\beta^2 \omega\left(\frac{X}{\beta R}\right)$$

WHERE:

R = VALUE OF r CORRESPONDING TO THE CONE SURFACE.

v = CROSS FLOW VELOCITY ($v = \alpha U$)

α = ANGLE OF ATTACK

U = AXIAL FLOW VELOCITY

AND WHERE THE FUNCTIONS $\Psi(n)$ AND $\omega(n)$ ARE:

$$\Psi(n) = n\sqrt{n^2-1} - \operatorname{arg\,cosh} n$$
$$\omega(n) = n\sqrt{n^2-1} + \operatorname{arg\,cosh} n$$

ON THE SURFACE OF THE CONE, $R = \delta X$, WHERE δ IS THE HALF CONE ANGLE.

WITH THE NOTATIONS:

$$n = X/\beta r$$
$$N = X/\beta R$$

IT IS FOUND:

$$\frac{W}{V} = -\frac{\Psi(n)}{\omega(N)}$$

THE RATIO W/V IS EQUAL TO $d\epsilon/d\alpha$, ϵ BEING THE DOWNWASH ANGLE WHICH CORRESPONDS TO α.

THUS:

$$\frac{d\epsilon}{d\alpha} = -\frac{\Psi(n)}{\omega(N)}$$

THE DOWNWASH ANGLE IS NEGATIVE, AND ACTUALLY IS AN UPWASH ANGLE, WHICH SHOWS THAT A BODY OF REVOLUTION MAY PRODUCE AN ADDITIONAL LIFT ON WINGS ATTACHED TO IT.

ON THE SURFACE OF THE CONE:

$$\frac{d\epsilon}{d\alpha} = -\frac{\Psi(N)}{\omega(N)} = -\frac{\Psi(1/\beta\delta)}{\omega(1/\beta\delta)}$$

ON THE SURFACE OF THE MACH CONE ORIGINATING AT THE APEX OF THE CONE, $n = 1$ AND $d\epsilon/d\alpha = 0$.

THE UPWASH OF A CONE INCREASES WHEN δ, THE HALF ANGLE OF THE CONE, IS DECREASED. FOR EXTREMELY SMALL VALUES OF $\beta\delta$ AND OF $\beta r/x$, THE TWO FUNCTIONS $\omega(N)$ AND $\Psi(n)$ TEND TOWARD N^2 AND n^2 RESPECTIVELY, SO THAT:

$$\frac{d\epsilon}{d\alpha} = \frac{n^2}{N^2} = \frac{R^2}{r^2}$$

THIS IS THE CONVENTIONAL RESULT FOR INCOMPRESSIBLE FLOW PAST A CYLINDER; THE VELOCITY DISTRIBUTION IN A PLANE PERPENDICULAR TO THE FLOW IS DEFINED BY THE LAW:

$$W + V = \left(1 + \frac{R^2}{r^2}\right) V$$

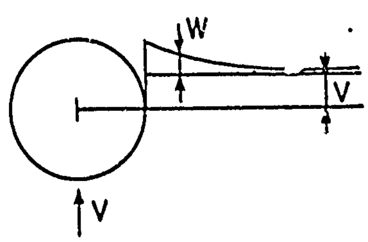

FIGURE 1

THIS SHOWS THE CONSIDERABLE INCREASE IN EFFECTIVENESS OF A WING ATTACHED TO A CYLINDRICAL FUSELAGE, IN A REGION OF THE FUSELAGE SUFFICIENTLY DISTANT FROM THE END CONE:

THE EFFECTIVE ANGLE OF ATTACK IS DOUBLED IN THE VICINITY OF THE FUSELAGE, AND IS INCREASED BY 50% AT A DISTANCE FROM THE FUSELAGE EQUAL TO 41% OF ITS DIAMETER. (FIG. 1)

GENERAL CASE:

IN THE GENERAL CASE, IT IS KNOWN THAT THE POTENTIAL ϕ_2 CAN BE WRITTEN IN FORM:

$$\phi_2 = \frac{1}{2}\beta^2 \cos\theta \; r \sum_{i=1}^{n} K_i \left[\Psi\left(\frac{x-\xi_{i-1}}{\beta r}\right) - \Psi\left(\frac{x-\xi_i}{\beta r}\right)\right]$$

WHERE K_i ARE THE DERIVATIVES OF THE DOUBLET DISTRIBUTIONS, THESE DERIVATIVES ARE ASSUMED TO BE CONSTANT IN THE CORRESPONDING INTERVALS (ξ_{i-1}, ξ_i)

WHERE:

$$\xi_i = x_i - \beta R_i$$

x_i, R_i BEING THE COORDINATES OF A POINT P_i ON THE MERIDIAN OF THE SURFACE, AND $i = n$ BEING SUCH THAT $x - \xi_n > 0$

THE POTENTIAL CAN BE PUT IN THE FOLLOWING FORM, WHICH IS MORE CONVENIENT FOR NUMERICAL COMPUTATIONS, IF A GREAT NUMBER OF POINTS ARE TO BE INVESTIGATED:

$$\phi_2 = \frac{1}{2}\beta^2 \cos\theta \; r\left[K_1\Psi\left(\frac{x}{\beta r}\right) + \sum_{i=1}^{i=n-1}(K_{i+1} - K_i)\Psi\left(\frac{x-\xi_i}{\beta r}\right) - K_n\Psi\left(\frac{x-\xi_n}{\beta r}\right)\right]$$

THE CONSTANTS K ARE DEFINED BY THE FOLLOWING RELATIONS ON THE SURFACE OF THE BODY:

$$v = \left(\frac{d\phi_2}{d_r}\right)_{\phi=0} = -\frac{1}{2}\beta^2 \sum_{i=1}^{n} K_i\left[\omega\left(\frac{x_n-\xi_{i-1}}{\beta R_n}\right) - \omega\left(\frac{x_n-\xi_i}{\beta R_n}\right)\right]$$

AND THE DOWNWASH COEFFICIENT $d\epsilon/d\alpha$ IS:

$$\frac{d\epsilon}{d\alpha} = -\frac{\phi_2/r}{d\phi_2/dr} = -\frac{\sum K_i\left\{\Psi\left(\frac{x-\xi_{i-1}}{\beta r}\right) - \Psi\left(\frac{x-\xi_i}{\beta r}\right)\right\}}{\sum_{i=1}^{n} K_i\left\{\omega\left(\frac{x_n-\xi_{i-1}}{\beta R_n}\right) - \omega\left(\frac{x_n-\xi_i}{\beta R_n}\right)\right\}}$$

IT IS ASSUMED THAT THE BODY HAS A CONICAL NOSE, THEN FOR THE NOSE REGION $V = \frac{1}{2}\beta^2 K_1 \omega\left(\frac{t}{\beta}\right)$, WHERE $t = X/R$ IS THE SLOPE OF THE CONE SURFACE (TANGENT OF HALF ANGLE)

THEN:

$$\frac{d\epsilon}{d\alpha} = -\frac{\Psi\left(\frac{x}{\beta r}\right) + \sum_{i=1}^{n}\left(\overline{K}_{i+1} - \overline{K}_i\right)\Psi\left(\frac{x-\xi_i}{\beta r}\right) - \overline{K}_n\Psi\left(\frac{x-\xi_n}{\beta r}\right)}{\omega\left(\frac{t}{\beta}\right)}$$

WHERE $\overline{K}_i$ ARE UNIT VALUES OF K_i, BASED ON $K_1 = 1$ AND DEFINED BY:

$$\omega\left(\frac{t}{\beta}\right) = \sum_{i=1}^{n}\overline{K}_i\left[\omega\left(\frac{x_n-\xi_{i-1}}{\beta R_n}\right) - \omega\left(\frac{x_n-\xi_i}{\beta R_n}\right)\right]$$

NUMERICAL RESULTS AND PRACTICAL CONCLUSIONS:

(A) CONE OF REVOLUTION

TABLE I GIVES THE VALUES OF THE FUNCTION $d\epsilon/d\alpha$:

$$-\frac{d\epsilon}{d\alpha} = +\frac{\Psi(n)}{\omega(N)}$$

THESE VALUES ARE PLOTTED IN CHART I. IT IS APPARENT THAT FOR SMALL CONE ANGLES (SMALL N VALUES), THE DOWNWASH DISTRIBUTION IS APPROXIMATELY GIVEN BY THE EXPRESSION N^2/n^2.

(b) General Case

Table II is an auxiliary table of the two functions $\omega(x)$ and $\psi(x)$:

$$\omega(x) = x\sqrt{x^2-1} + \arg\cosh x$$
$$\psi(x) = x\sqrt{x^2-1} - \arg\cosh x$$

It can be applied to the determination of the doublet distribution corresponding to a body of revolution at a small angle of attack. This application has been done for an ogival shaped body, with a cylindrical rear portion.

Table III indicates the shape of the body and summarizes the results of Tsien's method (doublet distribution).

Figure II shows the shape of the body of revolution, and the corresponding doublet distribution.

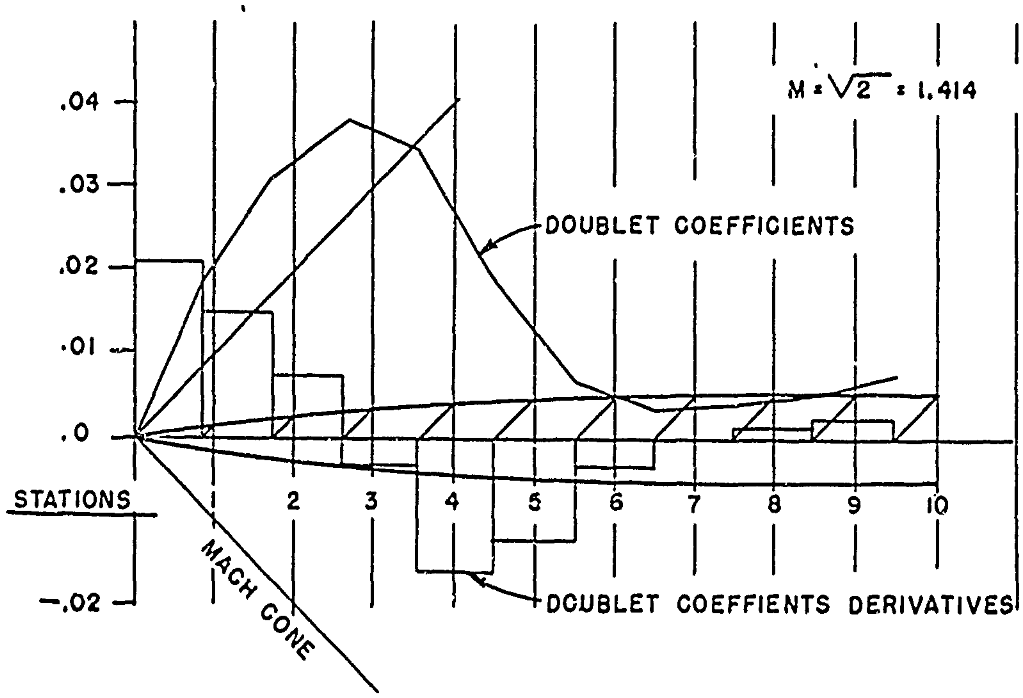

Figure II - Example of Upwash Analysis

Chart II shows the upwash distribution at several stations. The representation of the upwash given in this chart does not allow a simple comparison with the $1/r^2$ distribution and with the results established for cones. For this reason, two other charts have been plotted.

In Chart III the abscisses are the ratios r/R, instead of the values r used in Chart II The curve for the upwash around a cylinder is also shown, and it is

APPARENT THAT THIS CURVE IS GIVING A GOOD REPRESENTATION OF THE UPWASH DISTRIBUTION, EXCEPT IN THE FORWARD PORTION OF THE BODY, WHERE THE MACH CONE IS NOT DISTANT FROM THE SURFACE OF THE BODY.

IN CHART IV THE ABSCISSES ARE THE RATIOS $r/\beta X$, INSTEAD OF THE VALUES r. THE DOTTED LINES ARE CURVES TAKEN FROM CHART I, AND INDICATE VERY CLEARLY THAT THESE CURVES CAN BE USED FOR ALL PRACTICAL APPLICATIONS INSTEAD OF THE VALUES CALCULATED FOR THE OGIVE UNDER CONSIDERATION. IT IS THUS APPARENT THAT THE UPWASH DISTRIBUTION AT A STATION IS MAINLY A FUNCTION OF THE RATIO $R/\beta X$, I.E., OF THE RATIO OF THE RADIUS OF THE BODY TO THE RADIUS AT THE SAME STATION, OF THE MACH CONE ORIGINATING AT THE APEX OF THE BODY OF REVOLUTION. THIS STATEMENT SHOULD NOT BE CONSIDERED AS ABSOLUTELY GENERAL, BUT IS PROBABLY A GOOD INDICATION FOR DESIGN PURPOSES. IF THIS PRACTICAL CONCLUSION IS APPLIED, IT BECOMES UNNECESSARY, IN ORDER TO DETERMINE THE UPWASH DISTRIBUTION, TO KNOW THE DOUBLET DISTRIBUTION WHICH REPRESENTS THE BODY OF REVOLUTION. IT IS SUFFICIENT TO CALCULATE THE VALUES OF $N=R/\beta X$ AT THE REQUIRED STATIONS, AND TO USE THE CORRESPONDING CURVES IN CHART I, WHILE THE VALUES OF n ARE DEFINED AS $n=r/\beta X$. THIS DEFINES VERY RAPIDLY THE COMPLETE UPWASH DISTRIBUTION AROUND THE BODY.

GENERAL CONCLUSIONS:

THE UPWASH AROUND A SUPERSONIC BODY OF REVOLUTION IS AN IMPORTANT FACTOR IN THE DETERMINATION OF THE LIFT OF WING OR TAIL SURFACES ATTACHED TO THE BODY, ESPECIALLY IF SMALL SPANS ARE USED. IN THE VICINITY OF THE FUSELAGE, THE UPWASH MAY INCREASE THE SLOPE OF THE LEFT CURVE BY AS MUCH AS 100%, AND FOR SMALL SPANS, THE AVERAGE INCREASE OF THE SLOPE OF THE LEFT CURVE MAY BE OF THE ORDER OF 25% OR 50%.

VALUES OF THE UPWASH DISTRIBUTION AROUND CONES OF REVOLUTION ARE GIVEN IN TABLE I AND IN CHART I, AND NUMERICAL APPLICATION OF THE GENERAL METHOD OF ANALYSIS TO A BODY OF REVOLUTION SHOWS THAT THE VALUES FOR CONES CAN BE USED AS FIRST APPROXIMATION TO DETERMINE THE UPWASH AROUND OTHER SHAPES OF BODIES OF REVOLUTIONS, WITH THE PROBABLE EXCEPTION OF BODIES WITH HIGH NOSE AND CONE ANGLES.

TABLE I

VALUES OF $-d\epsilon/d\alpha = \Psi(n)/\Psi(N)$

$n = x/\beta r$; $N = X/\beta r = 1/\delta\beta$

n \ N	.9	.8	.7	.6	.5	.45	.4	.35	.3	.25	.2	.15	.1	.05
.9	.07023	.04302	.03078	.02104	.01467	.01202	.00962	.00750	.00563	.00399	.00262	.00151	.00068	.00017
.8		.14996	.10731	.07332	.05114	.04190	.03352	.02613	.01961	.01393	.00913	.00525	.00239	.00061
.7			.24766	.16923	.11804	.09671	.07736	.06031	.04525	.03215	.02107	.01213	.00551	.00140
.6				.33964	.23690	.19409	.15526	.12105	.09082	.06452	.04228	.02434	.01105	.00281
.5					.44909	.36794	.29433	.22947	.17217	.12231	.08016	.04614	.02095	.00533
.45						.50800	.40637	.31682	.23771	.16887	.11067	.06371	.02892	.00735
.40							.57045	.44474	.33369	.23705	.15535	.08943	.04060	.01032
.35								.63433	.47594	.33810	.22157	.12756	.05791	.01472
.30									.69951	.49691	.32566	.18748	.08511	.02164
.25										.76492	.50130	.28859	.13102	.03331
.20											.82884	.47715	.21663	.05507
.15												.88891	.40356	.10259
.1													.94159	.23936
.05														.98170

FUNCTIONS $\omega(x)$ AND $\Psi(x)$

x	ω(x)	Ψ(x)	x	ω(x)	Ψ(x)
1.0	0.00000	0.00000	1.80	3.88690	1.50108
1.02	0.40469	0.00535	1.82	3.97377	1.56143
1.04	0.57900	0.01518	1.84	4.06117	1.62273
1.06	0.71736	0.02796	1.86	4.14913	1.68499
1.08	0.83793	0.04317	1.88	4.23765	1.74819
1.10	0.94766	0.06052	1.90	4.32674	1.81234
1.12	1.05004	0.07978	1.92	4.41642	1.87744
1.14	1.14715	0.10083	1.94	4.50667	1.94347
1.16	1.24034	0.12354	1.96	4.59753	2.01043
1.18	1.33053	0.14783	1.98	4.68899	2.07833
1.20	1.41834	0.17362	2.00	4.78106	2.14714
1.22	1.50434	0.20088	2.02	4.87375	2.21689
1.24	1.58884	0.22952	2.04	4.96705	2.28755
1.26	1.67219	0.25951	2.06	5.06099	2.35913
1.28	1.75461	0.29083	2.08	5.15555	2.43163
1.30	1.83629	0.32343	2.10	5.25076	2.50504
1.32	1.91742	0.35728	2.12	5.34660	2.57936
1.34	1.99811	0.39235	2.14	5.44309	2.65457
1.36	2.07851	0.42863	2.16	5.54025	2.73071
1.38	2.15868	0.46608	2.18	5.63805	2.80775
1.40	2.23873	0.50470	2.20	5.73652	2.88568
1.42	2.31874	0.54446	2.22	5.83565	2.96451
1.44	2.39876	0.58536	2.24	5.93544	3.04424
1.46	2.47885	0.62735	2.26	6.03591	3.12487
1.48	2.55908	0.67044	2.28	6.13706	3.20638
1.50	2.63947	0.71463	2.30	6.23887	3.28879
1.52	2.72009	0.75989	2.32	6.34138	3.37208
1.54	2.80095	0.80621	2.34	6.44456	3.45626
1.56	2.88209	0.85357	2.36	6.54844	3.54132
1.58	2.96356	0.90198	2.38	6.65300	3.62726
1.60	3.04537	0.95143	2.40	6.75826	3.71410
1.62	3.12754	1.00190	2.42	6.86420	3.80180
1.64	3.21011	1.05339	2.44	6.97086	3.89040
1.66	3.29309	1.10589	2.46	7.07822	3.97988
1.68	3.37649	1.15939	2.48	7.18626	4.07020
1.70	3.46034	1.21388	2.50	7.29503	4.16143
1.72	3.54467	1.26937	2.52	7.40448	4.25350
1.74	3.62947	1.32583	2.54	7.51466	4.34648
1.76	3.71476	1.38330	2.56	7.62553	4.44029
1.78	3.80058	1.44170	2.58	7.73714	4.53500
1.80	3.88690	1.50108	2.60	7.84944	4.63056

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FUNCTIONS $\omega(x)$ AND $\Psi(x)$

X	$\omega(x)$	$\Psi(x)$	X	$\omega(x)$	$\Psi(x)$
2.60	7.84944	4.63056	3.40	12.94325	9.15413
2.62	7.96245	4.72699	3.42	13.08590	9.28452
2.64	8.07621	4.82429	3.44	13.22934	9.41576
2.66	8.19066	4.92244	3.46	13.37355	9.54785
2.68	8.30585	5.02149	3.48	13.51851	9.68077
2.70	8.42174	5.12136	3.50	13.66420	9.81450
2.72	8.53839	5.22213	3.52	13.81068	9.94910
2.74	8.65575	5.32373	3.54	13.95791	10.08451
2.76	8.77382	5.42620	3.56	14.10591	10.22077
2.78	8.89266	5.52954	3.58	14.25466	10.35784
2.80	9.01219	5.63371	3.60	14.40417	10.49575
2.82	9.13246	5.73876	3.62	14.55445	10.63451
2.84	9.25347	5.84465	3.64	14.70549	10.77409
2.86	9.37521	5.95141	3.66	14.85731	10.91451
2.88	9.49767	6.05901	3.68	15.00990	11.05576
2.90	9.62089	6.16747	3.70	15.16321	11.19781
2.92	9.74485	6.27679	3.72	15.31732	11.34072
2.94	9.86954	6.38696	3.74	15.47218	11.48446
2.96	9.99496	6.49796	3.76	15.62781	11.62903
2.98	10.12114	6.60984	3.78	15.78425	11.77445
3.00	10.24804	6.72254	3.80	15.94140	11.92066
3.02	10.37567	6.83609	3.82	16.09935	12.06773
3.04	10.50407	6.95051	3.84	16.25806	12.21562
3.06	10.63322	7.06576	3.86	16.41751	12.36431
3.08	10.76309	7.18185	3.88	16.57776	12.51386
3.10	10.89373	7.29881	3.90	16.73879	12.66425
3.12	11.02510	7.41660	3.92	16.90054	12.81542
3.14	11.15723	7.53525	3.94	17.06308	12.96744
3.16	11.29011	7.65473	3.96	17.22642	13.12032
3.18	11.42374	7.77506	3.98	17.39049	13.27397
3.20	11.55811	7.89623	4.00	17.55536	13.42848
3.22	11.69321	8.01823	4.04	17.88740	13.73998
3.24	11.82909	8.14107	4.08	18.22252	14.05476
3.26	11.96570	8.26476	4.12	18.56076	14.37288
3.28	12.10310	8.38930	4.16	18.90205	14.69427
3.30	12.24125	8.51469	4.20	19.24647	15.01897
3.32	12.38014	8.64090	4.24	19.59395	15.34695
3.34	12.51979	8.76797	4.28	19.94454	15.67822
3.36	12.66018	8.89584	4.32	20.29825	16.01279
3.38	12.80132	9.02456	4.36	20.65505	16.35065
3.40	12.94325	9.15413	4.40	21.01495	16.69179

FUNCTIONS ω(x) AND Ψ(x)

X	ω(x)	Ψ(x)	X	ω(x)	Ψ(x)
4.40	21.01495	16.69179	6.00	37.97437	33.01859
4.44	21.37796	17.03622	6.04	38.46275	33.49349
4.48	21.74408	17.38394	6.08	38.95429	33.97165
4.52	22.11331	17.73493	6.12	39.44899	34.45305
4.56	22.48566	18.08922	6.16	39.94683	34.93769
4.60	22.86112	18.44678	6.20	40.44783	35.42557
4.64	23.23969	18.80761	6.24	40.95198	35.91670
4.68	23.62139	19.17173	6.28	41.45931	36.41107
4.72	24.00620	19.53912	6.32	41.96978	36.90868
4.76	24.39413	19.90979	6.36	42.48341	37.40953
4.80	24.78518	20.28372	6.40	43.00020	37.91362
4.84	25.17936	20.66094	6.44	43.52014	38.42096
4.88	25.57667	21.04141	6.48	44.04324	38.93152
4.92	25.97709	21.42517	6.52	44.56951	39.44533
4.96	26.38064	21.81218	6.56	45.09894	39.96238
5.00	26.78733	22.20247	6.60	45.63153	40.48267
5.04	27.19714	22.59602	6.64	46.16727	41.00619
5.08	27.61009	22.99283	6.68	46.70617	41.53295
5.12	28.02617	23.39291	6.72	47.24825	42.06295
5.16	28.44538	23.79626	6.76	47.79348	42.59618
5.20	28.86772	24.20286	6.80	48.34188	43.13264
5.24	29.29320	24.61272	6.84	48.89344	43.67234
5.28	29.72181	25.02585	6.88	49.44817	44.21529
5.32	30.15357	25.44223	6.92	50.00607	44.76147
5.36	30.58846	25.86188	6.96	50.56712	45.31086
5.40	31.02650	26.28478	7.00	51.13134	45.86350
5.44	31.46767	26.71093	7.04	51.69872	46.41938
5.48	31.91198	27.14034	7.08	52.26928	46.97848
5.52	32.35945	27.57301	7.12	52.84300	47.54082
5.56	32.81005	28.00893	7.16	53.41987	48.10639
5.60	33.26379	28.44811	7.20	53.99993	48.67519
5.64	33.72069	28.89053	7.24	54.58315	49.24723
5.68	34.18072	29.33620	7.28	55.16954	49.82248
5.72	34.64391	29.78513	7.32	55.75911	50.40099
5.76	35.11023	30.23731	7.36	56.35183	50.98271
5.80	35.57971	30.69273	7.40	56.94772	51.56766
5.84	36.05235	31.15141	7.44	57.54679	52.15585
5.88	36.52812	31.61334	7.48	58.14903	52.74727
5.92	37.00706	32.07852	7.52	58.75443	53.34191
5.96	37.48913	32.54693	7.56	59.36301	53.93977
6.00	37.97437	33.01859	7.60	59.97475	54.54087

FUNCTIONS ω(x) AND Ψ(x)

X	ω(x)	Ψ(x)	X	ω(x)	Ψ(x)
7.60	59.97475	54.54087	9.20	87.04789	81.22913
7.64	60.58967	55.14521	9.24	87.78988	81.96238
7.68	61.20777	55.75277	9.28	88.53502	82.69884
7.72	61.82902	56.36354	9.32	89.28337	83.43853
7.76	62.45345	56.97755	9.36	90.03490	84.18144
7.80	63.08107	57.59479	9.40	90.78960	84.92756
7.84	63.71184	58.21526	9.44	91.54748	85.67690
7.88	64.34579	58.83895	9.48	92.30854	86.42946
7.92	64.98292	59.46586	9.52	93.07278	87.18524
7.96	65.62322	60.09600	9.56	93.84021	87.94423
8.00	66.26669	60.72937	9.60	94.61082	88.70644
8.04	66.91334	61.36596	9.64	95.38461	89.47187
8.08	67.56316	62.00578	9.68	96.16158	90.24052
8.12	68.21615	62.64883	9.72	96.94175	91.01239
8.16	68.87233	63.29509	9.76	97.72507	91.78747
8.20	69.53167	63.94459	9.80	98.51161	92.56577
8.24	70.19419	64.59729	9.84	99.30131	93.34729
8.28	70.85988	65.25324	9.88	100.09419	94.13201
8.32	71.52876	65.91240	9.92	100.89027	94.91997
8.36	72.20[illegible]	66.57479	9.96	101.68952	95.71114
8.40	72.87604	67.24040	10.00	102.49196	96.50552
8.44	73.55444	67.90922	10.10	104.51196	98.50552
8.48	74.23601	68.58129	10.20	106.55189	100.52556
8.52	74.92077	69.25657	10.30	108.61172	102.56580
8.56	75.60870	69.93506	10.40	110.69144	104.62620
8.60	76.29980	70.61678	10.50	112.79118	106.70670
8.64	76.99409	71.30173	10.60	114.91102	108.80750
8.68	77.69156	71.98990	10.70	117.05054	110.92807
8.72	78.39221	72.68129	10.80	119.20962	113.06844
8.76	79.09603	73.37589	10.90	121.38873	115.22913
8.80	79.80302	74.07372	11.00	123.58792	117.40998
8.84	80.51321	74.77477	11.10	125.80700	119.61090
8.88	81.22656	75.47904	11.20	128.04601	121.83190
8.92	81.94311	76.18653	11.30	130.30493	124.07299
8.96	82.66282	76.89724	11.40	132.58379	126.33415
9.00	83.38572	77.61118	11.50	134.88255	128.61547
9.04	84.11179	78.32833	11.60	137.20110	130.91690
9.08	84.84106	79.04870	11.70	139.53971	133.23833
9.12	85.57348	79.77228	11.80	141.89836	135.57975
9.16	86.30910	80.49910	11.90	144.27698	137.94122
9.20	87.04789	81.22913	12.00	146.67542	140.32282

FUNCTIONS $\omega(x)$ AND $\Psi(x)$

X	$\omega(x)$	$\Psi(x)$	X	$\omega(x)$	$\Psi(x)$
12.00	146.67542	140.32282	16.00	258.96422	252.03472
12.10	149.09379	142.72443	16.10	262.18045	255.23845
12.20	151.53208	145.14622	16.20	265.41664	258.46222
12.30	153.99027	147.58806	16.30	268.67279	261.70607
12.40	156.46837	150.04995	16.40	271.94897	264.96997
12.50	158.96639	152.53187	16.50	275.24509	268.25392
12.60	161.48440	155.03390	16.60	278.56114	271.55788
12.70	164.02242	157.55606	16.70	281.89715	274.88185
12.80	166.58032	160.09825	16.80	285.25316	278.22590
12.90	169.15813	162.66044	16.90	288.62917	281.59001
13.00	171.75584	165.24264	17.00	292.02510	284.97414
13.10	174.37348	167.84490	17.10	295.44101	288.37829
13.20	177.01111	170.46727	17.20	298.87685	291.80245
13.30	179.66875	173.10977	17.30	302.33264	295.24567
13.40	182.34633	175.77238	17.40	305.80843	298.71091
13.50	185.04374	178.45494	17.50	309.30415	302.19516
13.60	187.76114	181.15752	17.60	312.81987	305.69945
13.70	190.49847	183.88016	17.70	316.35552	309.22372
13.80	193.25578	186.62285	17.80	319.91110	312.76800
13.90	196.03305	189.38559	17.90	323.48668	316.33236
14.00	198.83025	192.16839	18.00	327.08234	319.91676
14.10	201.64736	194.97126	18.10	330.69793	323.52125
14.20	20[illegible].48444	197.79418	18.20	334.33348	327.14580
14.30	207.34148	200.63714	18.30	337.98898	330.79038
14.40	210.21847	203.50015	18.40	341.66445	334.45497
14.50	213.11541	206.38323	18.50	345.35986	338.13957
14.60	216.03230	209.28636	18.60	349.07523	341.84409
14.70	218.96915	212.20949	18.70	352.81054	345.56864
14.80	221.92596	215.15269	18.80	356.56576	349.31326
14.90	224.90273	218.11599	18.90	360.34101	353.07801
15.00	227.89948	221.09937	19.00	364.13661	356.86285
15.10	230.91617	224.10271	19.10	367.95191	360.66763
15.20	233.95281	227.12611	19.20	371.78714	364.49240
15.30	237.00942	230.16956	19.30	375.64231	368.33717
15.40	240.08595	233.23305	19.40	379.51742	372.20192
15.50	243.18244	236.31658	19.50	383.41252	376.08674
15.60	246.29888	239.42013	19.60	387.32763	379.99159
15.70	249.43526	242.54370	19.70	391.26273	383.91651
15.80	252.59161	245.68732	19.80	395.21783	387.86147
15.90	255.76794	248.85100	19.90	399.19293	391.82649
16.00	258.96422	252.03472	20.00	403.18805	395.81155

TABLE III
(SEE FIGURE II)
EXAMPLE OF UPWASH ANALYSIS
M=1.414

(1) X	(2) R	(3) ξ	(4) $\frac{\beta^2}{2U}\frac{dK}{di}$	(5) $\frac{\beta^2}{2U}\frac{dW}{di}$	(6) $\frac{\beta}{4}\frac{dC_p}{di}$
1	.149	0.851	.0212	.0180	.141
2	.282	1.718	.0149	.0310	.124
3	.392	2.608	.0077	.0378	.105
4	.469	3.531	-.0035	.0346	.072
5	.500	4.500	-.0162	.0189	.025
6	.513	5.487	-.0121	.0069	.003
7	.524	6.476	-.0034	.0036	.003
8	.534	7.466	.0006	.0042	.008
9	.540	8.460	.0011	.0053	.004
10	.540	9.460	.0026	.0079	-.001

NOTES:

(1) X = STATION

(2) R = RADIUS AT STATION

(3) ξ = STATION FOR DOUBLET DISTRIBUTION

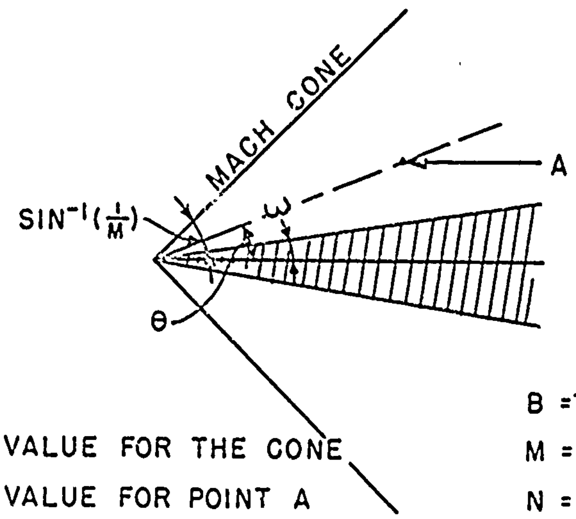
(4) K = DOUBLET STRENGTH COEFFICIENT DERIVATIVE

(5) W = DOUBLET STRENGTH COEFFICIENT

(6) dC_p/di = PRESSURE COEFFICIENT DERIVATIVE (FOR MAXIMUM LOCAL PRESSURE AT A STATION)

CHART I

UPWASH AROUND A CONE AT SMALL ANGLES OF ATTACK



UPWASH RATIO AT A:

DETERMINE
(1) N VALUE FOR THE CONE
(2) n VALUE FOR POINT A
(3) ENTER CURVE n
(4) READ ORDINATE $-d\epsilon/d\alpha$ CORRESPONDING TO n

$B = \sqrt{M^2 - 1}$
M = MACH NUMBER
$N = \beta \cdot \tan \omega$
$n = \beta \cdot \tan \theta$
ω = ½ CONE ANGLE
ε = UPWASH ANGLE
α = ANGLE OF ATTACK

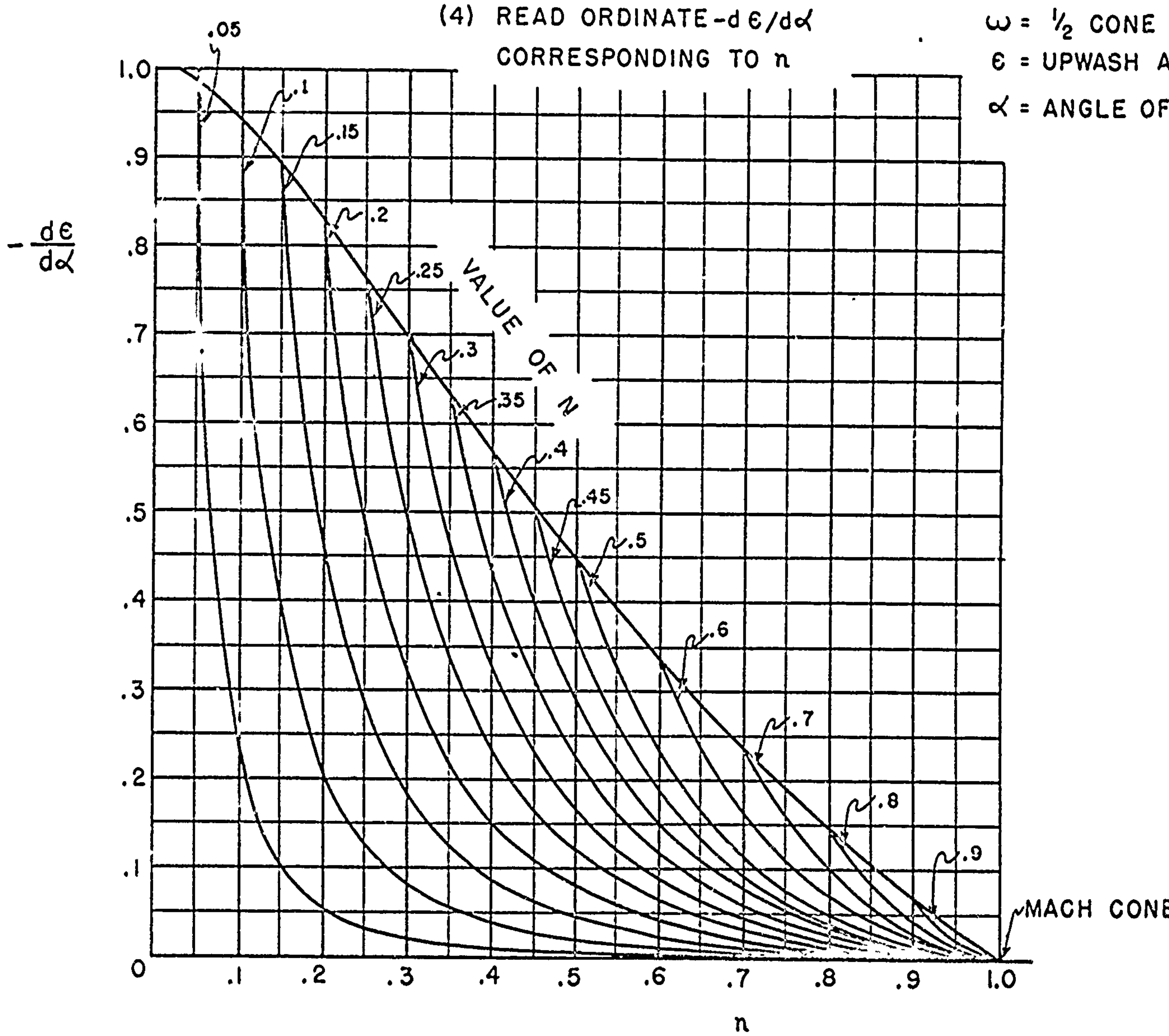


CHART II

UPWASH DISTRIBUTION FOR THE

BODY OF REVOLUTION SHOWN IN FIGURE II

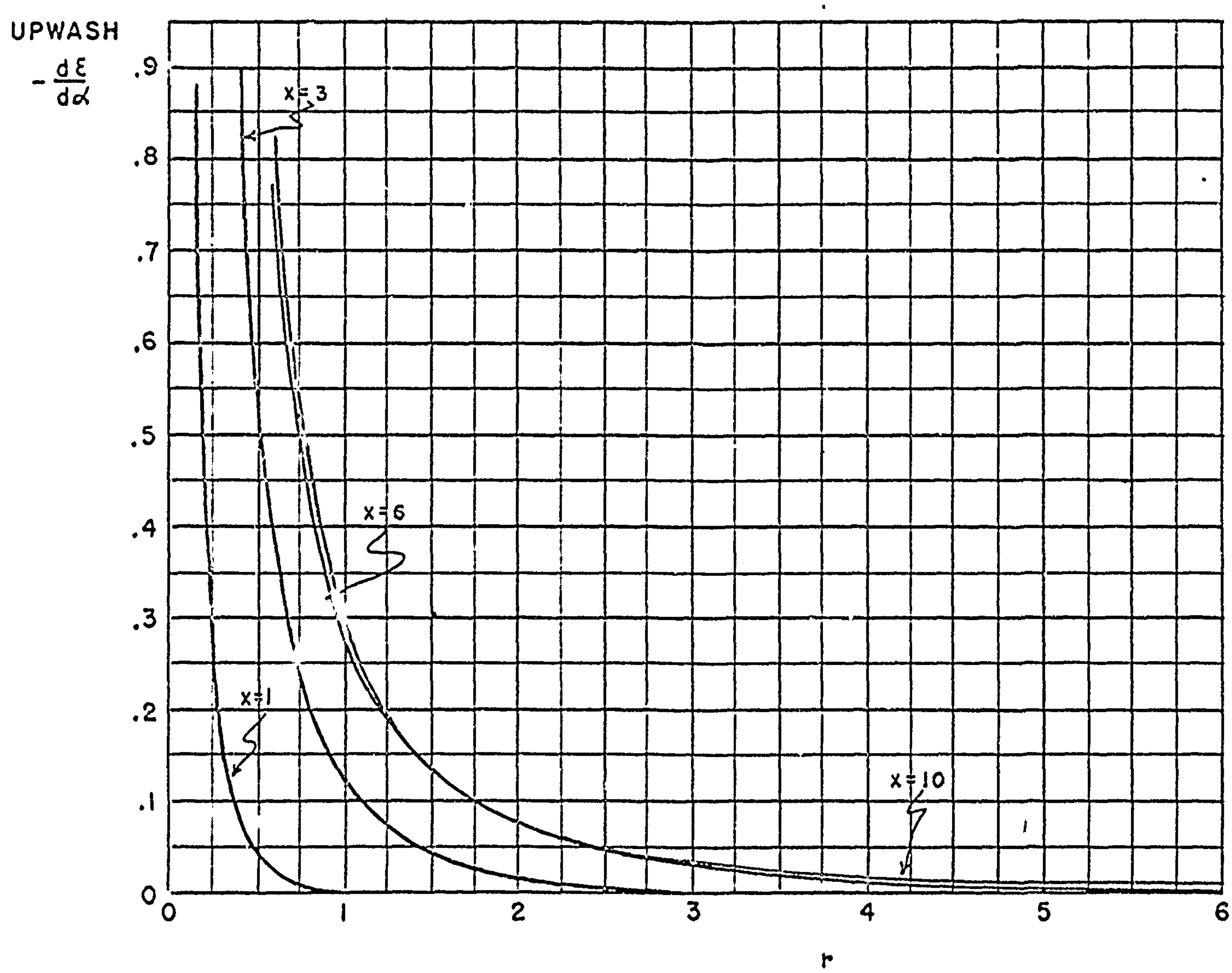


CHART III

UPWASH DISTRIBUTION FOR THE BODY

OF REVOLUTION SHOWN IN FIGURE II

COMPARISON WITH THE UPWASH OF A CYLINDER

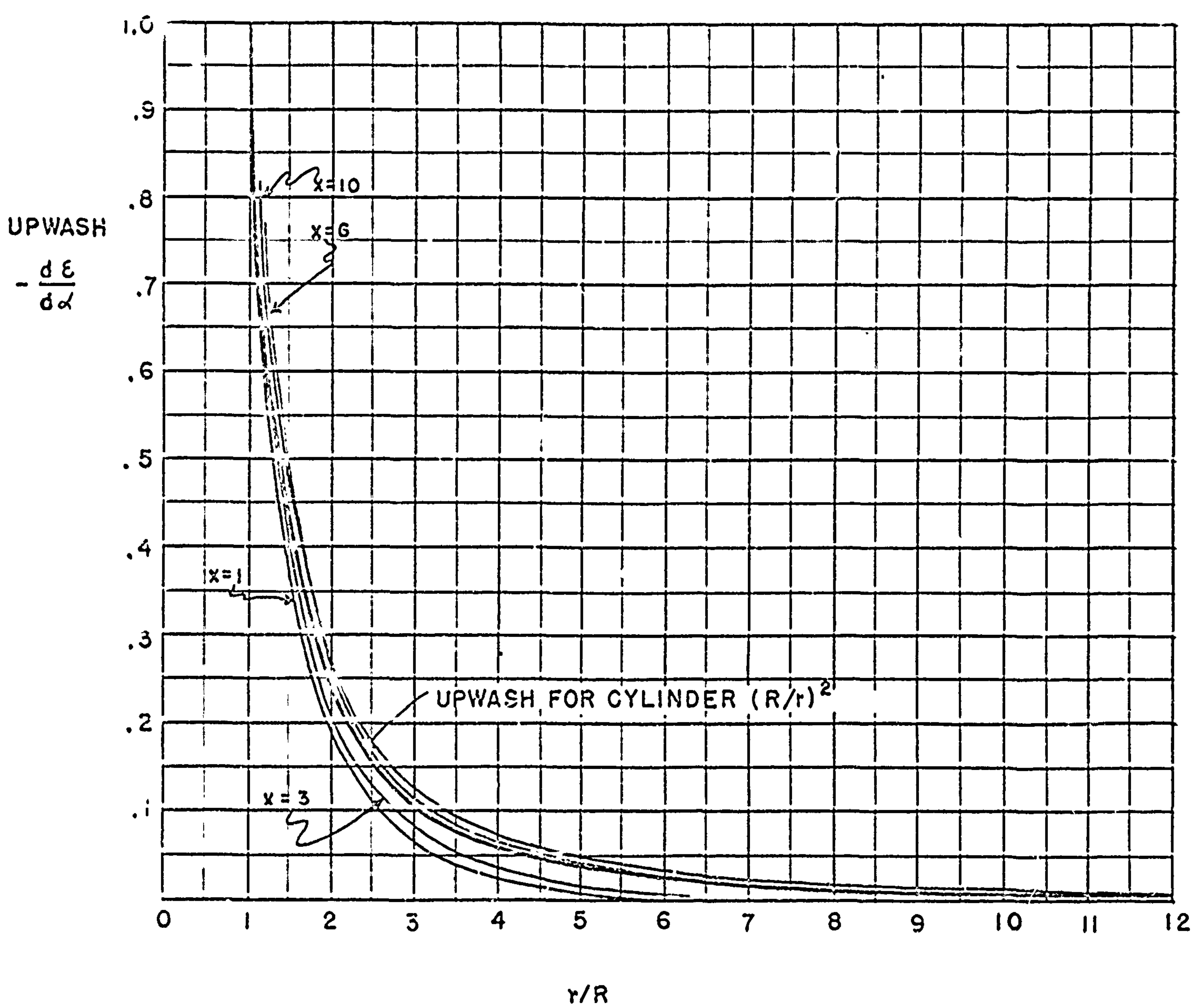
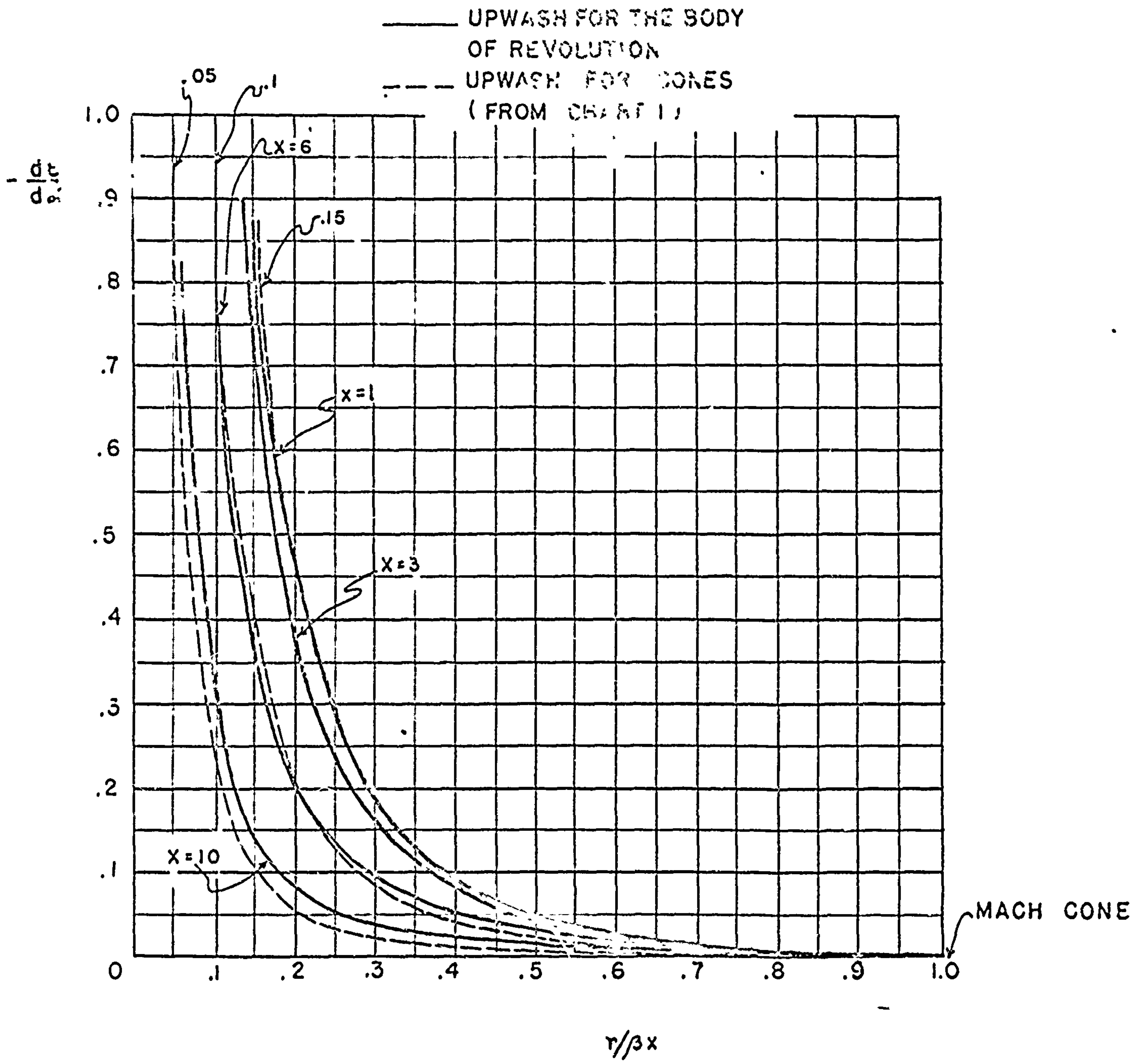


CHART IV

UPWASH DISTRIBUTION FOR THE BODY OF REVOLUTION SHOWN IN FIGURE II

COMPARISON WITH UPWASHES OF CONES HAVING THE SAME APEX AS THE BODY OF REVOLUTION, AND THE SAME RADIUS AT A GIVEN SECTION



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ABSTRACT

Application is made of H. S. Tsien's formulas for the potential flow around bodies of revolution at supersonic velocities. Consideration is given the derivation of formulas, application to conical and cylindrical fuselages, general case, and numerical results. Conclusions are that upwash around a supersonic body of revolution is an important factor in determining the lift of attached wing or tail surfaces. Near the fuselage upwash may increase the slope of the lift curve 100 percent. For small spans the average slope increase of the lift curve may be 25 to 50 percent.